Required knowledge in mathematics

1 Algebra

1.1 Set theory. Operations on sets, characteristic functions. Maps, injectivity, surjectivity. Direct and inverse image of a set. Integers, finite sets, countability.

1.2 Real numbers, complex numbers, complex exponential. Application to plane geometry and to trigonometry. Polynomials, relations between the roots and the coefficients. Elementary arithmetics in $\mathbb{Z}/n\mathbb{Z}$. Order in $\mathbb{R}$. Maximum, minimum, supremum, infimum.

1.3 Finite dimensional vector spaces. Free families, generating families, bases, dimension. Determinant of n vectors, characterization of bases. Matrices, operations on matrices (product, inverse, rank, etc.). Determinant and trace of a square matrix. Linear map, matrix associated with a linear map. Linear systems of equations.

1.4 Diagonalization of matrices. Eigenvalues, eigenvectors of a square matrix. Stable subspaces. Similar matrices, geometrical interpretation. Characteristic polynomial, minimal polynomial, Cayley-Hamilton theorem.

1.5 Euclidean spaces, Euclidean geometry, scalar product, Cauchy-Schwarz inequality. Norms and associated distances. Euclidean spaces of finite dimension, orthonormal bases, orthogonal projections. Orthogonal matrices. Diagonalization of symmetric real matrices.

2 Analysis and differential geometry

2.1 Finite dimensional normed vector spaces. Equivalence of norms, open and closed sets, open balls and closed balls. Convergent sequences in normed vector spaces, continuous mappings. Compact sets, images of compact sets by continuous mappings, existence of extrema.

2.2 Real or complex valued functions defined on an interval. Derivative at a point, functions of class C^k. Mean value theorem, Taylor's formula. Primitive of continuous functions. Usual functions (exponential, logarithm, trigonometric functions, rational fractions). Sequences and series of functions, simple and uniform convergence.

2.3 Integration over a bounded interval. Integral of piecewise continuous functions. Fundamental theorem of calculus. Integration by parts, change of variables. Integrals depending on a parameter. Continuity under the sign $\int$, differentiation under the sign $\int$. Cauchy-Schwarz inequality.

2.4 Series of real or complex numbers, simple and absolute convergence. Integral comparison criterion. Product of absolutely convergent series. Power series, radius of convergence. Functions that can be expanded as power series on an interval. Taylor series expansion of usual functions: $\exp(t)$, $\sin(t)$, $\cos(t)$, $\ln(1+t)$, $(1+t)^a$ where a is a real number.

2.5 Differential equations. Linear scalar equations of order 1 or 2. Homogeneous equation, method of separation of variables. Particular solution, variation of parameters. Initial-value problem. Linear systems with constant coefficients. Notions on nonlinear differential equations.

2.6 Functions of several real variables. Partial derivatives, differential of a function defined on $\mathbb{R}^n$. Chain rule, functions of class C^k. Schwarz theorem. Diffeomorphisms, inverse function theorem, implicit function theorem. Critical points, local and global extrema.

Double integrals. Computation of double integrals by Fubini's theorem or by a change of variables (e.g. use of polar coordinates).

Plane curve, tangent vector at a point, length of a plane curve.

2.7 Probability, conditional probability. Independent events. Random variable, expectation. Gaussian random variable, mean, variance.

2.8 Advanced analysis. Lebesgue integral, dominated convergence theorem, monotone convergence, Fatou's lemma. Fubini's theorem for abstract spaces, change of variables. Riemann-Lebesgue lemma. Fourier series, Dirichlet theorem, Parseval theorem. Fourier transform, Plancherel theorem. Complex analytic functions, Laurent series, integral along a curve, Cauchy theorem.